A Story of Units is a curriculum written by teachers for teachers to help every student build mastery of the Common Core Learning Standards for Mathematics. The theme of the story - creating, manipulating, and relating units - glues seemingly separate ideas into a coherent whole throughout each grade and over the years.

As noted earlier in The Common Core Approach to Instructional Shifts, coherence is supported in A Story of *Units* through the use of a finite set of concrete and pictorial models. Students build increasing dexterity with these models through persistent use within and across levels of curriculum. The repeated appearance of familiar models helps to build the imperative vertical links between the topics of one grade level and the next. In addition, the depth of awareness that students have with the models not only ensures that they naturally become a part of the students' schema, but also facilitates a more rapid and multifaceted understanding of new concepts as they are introduced.

This information is designed to support teachers as they engage students in meaningful mathematical learning experiences aligned to the Common Core Learning Standards. This support is provided through the following information:

- The grade levels for which the model is most appropriate
- A description and example of the model
- A collection of instructional strategies for using the model presented in order of the natural progression of the concept(s)

The following categories indicate the primary application area for each model. However, as previously stated, models appear repeatedly across grades and topics. Therefore, instructional strategies will include examples spanning several levels of the curriculum.

Numbers Through 10

- Number Towers
- Number Path
- Number Bond

Place Value and Standard Algorithms

- Bundles
- Place Value Chart
- Base-Ten Blocks
- Money
- Number Disks (with Place Value Chart)

Fractions

- Number Line
- Area Model

Addition and Subtraction

- Ten-Frame

Multiplication

- Array and Area Model
- Rekenrek

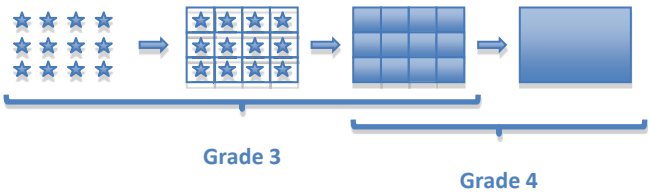
Word Problems

- Tape Diagram

Array and Area Models

Grade Level **1 – 5**

Description

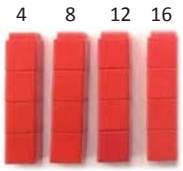


An array is an arrangement of a set of objects organized into equal groups in rows and columns. Arrays help make counting easy. Counting by equal groups is more efficient than counting objects one by one. The ten-frame is an array used in Kindergarten. Students count objects in arrays in Kindergarten and Pre-Kindergarten. (PK.CC.4) The rectangular array is used to teach multiplication and leads to understanding area. (3.OA.3)

Arrays reinforce the meaning of multiplication as repeated addition (e.g., 3 x 4 = 4 + 4 + 4), and the two meanings of division—that 12 ÷ 3 can indicate how many will be in each group if I make 3 equal groups and that it can also indicate how many groups I can make if I put 3 in each group. Further using arrays reinforces the relationship between multiplication and division.

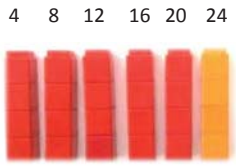
Instructional Strategies

- Use number towers to depict multiplication problems in the shape of an array.

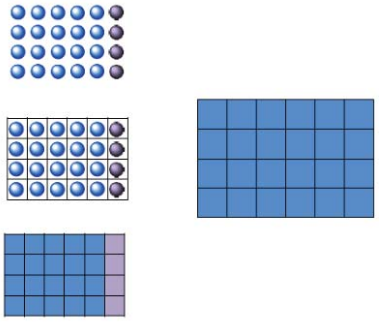


4 × 4 = ___.
16 ÷ 4 = __.

5 fours + 1 four = 6 fours
20 + 4 = 24
6 × 4 is 4 more than 5 × 4.



- Use the rectangular grid to model multiplication and division.



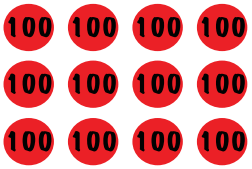
4 x 6 = ___
6 x 4 = ___

4 x ___ = 24
___ x 6 = 24

24 ÷ 4 = __
24 ÷ 6 = __

- Multiply units with arrays.

Multiplying hundreds:
4 hundreds × 3 = 12 hundreds
400 × 3 = 1200

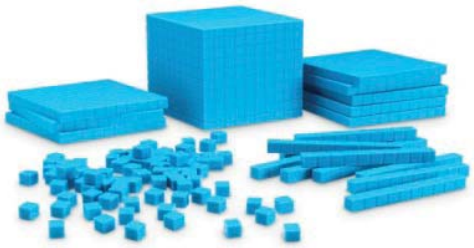


400 x 3 = ▢

Base-Ten Blocks

Grade Level **K – 2**

Description



Base-ten blocks (also referred to as Dienes blocks) include thousands "cubes," hundreds "flats," tens "rods," and ones. Base-ten blocks are a proportional representation of units of ones, tens, hundreds, and thousands and are useful for developing place value understanding. This is a "pre-grouped" model for base-ten that allows for more efficient modeling of larger quantities through the thousands. However, because this place value model requires students to more abstractly consider the 10 to 1 relationship of the various blocks, care must be taken to ensure that students attend to the "ten-ness" of the pieces that are now traded rather than bundled or un-bundled.

Base-ten blocks are introduced after students have learned the value of hundreds, tens, and ones and have had repeated experiences with composing and decomposing groups of 10 ones or groups of 10 tens with bundles.

Instructional Strategies

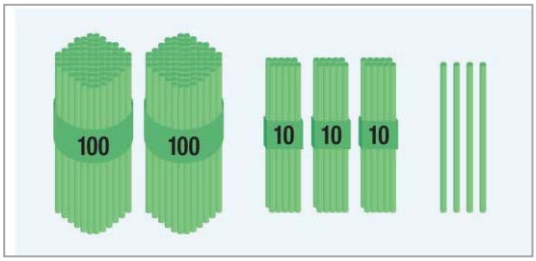
Please note: Instructional strategies for base-ten blocks are similar to those of bundles and place value disks. Therefore, see "Bundles," "Money," and "Place Value Disks" for other teaching ideas.

- Represent quantities on the mat and write in standard, expanded, and word form.
- Play "More" and "Less" games. Begin with an amount on a mat. At a predetermined signal (e.g., teacher claps or rings a bell), students add (or subtract) a quantity (2, 5, 10, or other) to the blocks on the mat.
- Give student equivalent representation riddles to be solved with base-ten pieces. For example, I have 29 ones and 2 hundreds. What number am I?
- Model addition, subtraction, multiplication, and division.
- Use blocks and mats as a support for teaching students to record the standard algorithms for all four operations.

Bundles

Grade Level **K – 2**

Description

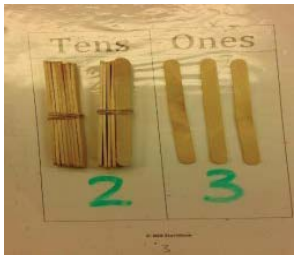


Bundles are discrete groupings of place value units (tens, hundreds, thousands), usually made by students/teachers placing a rubber band or chenille stem around straws, popsicle sticks, or coffee stirrers. Linking cubes may also be used in this fashion. Ten straws (or cubes) are bundled (or linked) into 1 unit of ten, 10 tens are bundled into 1 unit of a hundred, and so on. These student-made groupings provide the necessary conceptual foundation for children to be successful with pre-grouped, proportional, and non-proportional base-ten materials. (See Base-Ten Blocks and Number Disks.)

Understanding tens and ones is supported in Kindergarten as students learn to compose and decompose tens and ones by "bundling" and "unbundling" the materials. Numbers 11-19 are soon seen as 1 ten (a bundled set of 10 ones) and some extra ones.

By Grade 2, students expand their skill with and understanding of units by bundling units of ones, tens, and hundreds up to one thousand with sticks. These larger units are discrete and can be counted: "1 hundred, 2 hundred, 3 hundred, etc." Bundles also help students extend their understanding of place value to 1000. (2.NBT.1) Repeated bundling experiences help students to internalize the pattern that 10 of one unit make 1 of the next larger unit. Expanded form, increased understanding of skip-counting (2.NBT.2), and fluency in counting larger numbers are all supported by the use of this model.

Bundles are also useful in developing conceptual understanding of renaming in addition and subtraction. The mat below shows 2 tens and 3 ones. To solve 23 – 9, one bundle of ten is "unbundled" to get 1 ten and 13 ones in order to take away 9 ones.



Instructional Strategies

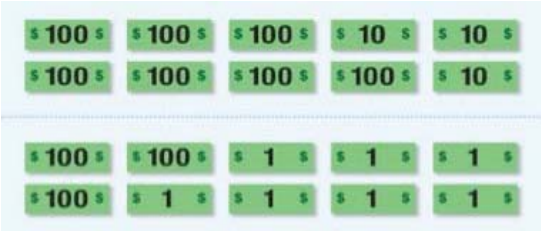
- Represent various quantities with bundles and "singles."
- Count school days. Each day a single straw/stick is added to the ones pocket and counted. Sticks are bundled when 10 days have passed and moved to the tens pocket. Have a "100th Day" celebration.

- Bundles may also be used to count down to a significant event (e.g.., the last day of school), unbundling as necessary.
- Play "Race to Zero" with a partner. Students start with a quantity between 30 and 40 in bundles. Roll two dice to determine what can be taken away from the starting quantity (unbundling as necessary). First partner to reach zero is the winner. (This game may also be played as an addition game.)
- Count in unit form (2 tens, 8 ones; 2 tens, 9 ones; 3 tens, etc.).
- Represent quantities on place value mats to be added or subtracted.

Money

Grade Level **2**

Description



Dollar bills (1s, 10s, and 100s) are non-proportional units that are used to develop place value understanding. That is, bills are an abstract representation of place value because their value is not proportionate to their size. Ten bills can have a value of $10 or $1000 but appear identical aside from their printed labels. Bills can be "traded" (e.g., 10 ten-dollar bills for 1 hundred-dollar bill) to help students learn equivalence of the two amounts.

As with other place value models, students can use bills to model numbers up to three digits, to read numbers formed with the bills, and to increase fluency in skip-counting by tens and hundreds.

The picture above shows that the arrangement of the $100s, $10s, and $1s can be counted in this manner:

The first frame, S: 100, 200, 300, 400, 500, 600, 700, 710, 720, 730.

The second frame, S: 100, 200, 300, 301, 302, 303, 304, 305, 306, 307.

The transition from a discrete unit of a "bundle" to proportional materials such as base-ten blocks to a non-proportional unit of a bill is a significant leap in a student's place value learning trajectory.

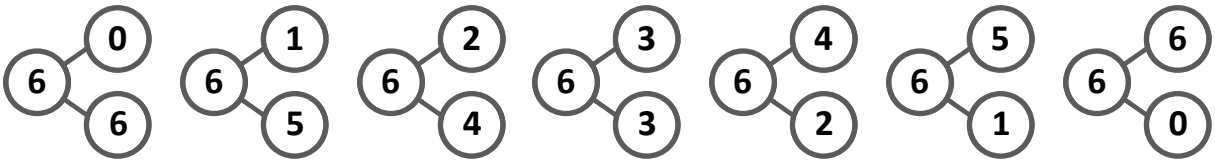
Instructional Strategies

- Skip-count up and down by $10 between 45 and 125.
  (45, 55, 65, 75, 85, 95, 105, 115, 125).
- Practice "making change" by counting on from an amount up to a specified total.
- "More" and "Less" games may also be played with money (See Base-Ten Blocks).
- Play equivalency games. How many $5 bills in a $10 bill? A $20 bill? A $100 bill? etc.

Number Bond

Grade Level K – 5

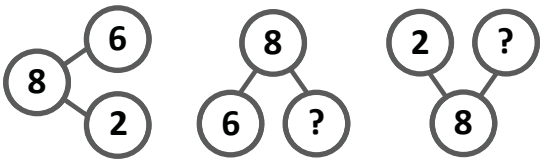
Description



The number bond is a pictorial representation of part-part-whole relationships and shows that within a part-whole relationship, smaller numbers (the parts) make up larger numbers (the whole). The number bond may be presented as shown, using smaller circles (or squares) for the parts to distinguish the part from the whole. As students become more comfortable using number bonds, they may be presented using the same size shape for parts and whole.

Number bonds of 10 have the greatest priority because students will use them for adding and subtracting across 10. Students move towards fluency in Grade 1 with numbers to 10 building on the foundation laid in Kindergarten. They learn to decompose numbers to ten with increasing fluency. (1.OA.6) Students learn the meaning of addition as "putting together" to find the whole or total and subtraction as "taking away" to find a part.

Notice in the diagrams below that the orientation of the number bond does not change its meaning and function. (6 + 2 = 8, 2 + 6 = 8, 8 – 6 = 2, 8 – 2 = 6)



Instructional Strategies

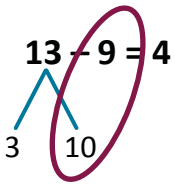
- Make bonds with a specified whole using concrete objects. Students place all the objects into the "parts" circles of the bond using various combinations. These can be recorded pictorially (students draw objects in the bonds), abstractly (children write numerals in the bonds), or a combination of these representations as appropriate.
- Generate number stories for each number from 5 to 10 from pictures and situations.
- Develop fluency: Show all the possible ways to make ___, for all the numbers from 1 to 10.
- Present bonds in which the whole and one part are visible (using concrete, pictorial, and eventually abstract representations). Students solve for the other part by bonding, counting on, or subtracting.
- Transition students from number bonds to tape diagrams by drawing both representations for number stories.
- Use number bonds as a support for mental math techniques such as "Make 10" (see grade specific

examples below).

- Use number bonds to see part-whole fraction and decimal relationships.

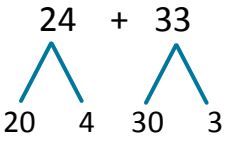
Grade 1 Example

Decompose 13 into 10 and 3.
Subtract 9 from the 10.
10 – 9 = 1
Then add 1 + 3.
1 + 3 = 4, so 13 – 9 = 4



Grade 2 Example

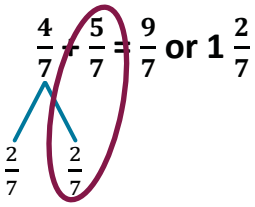
Solve 24 +33 mentally.
Use bonds to show your thinking.



(20 + 30) + (4 + 3) = 57

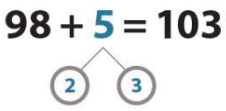
Grade 4 Example 1

Decompose $\frac{4}{7}$ into $\frac{2}{7}$ and $\frac{2}{7}$.
Add $\frac{2}{7}$ to $\frac{5}{7}$ to make 1 whole.
$\frac{2}{7} + \frac{5}{7} = \frac{7}{7}$
Then add $\frac{7}{7}$ to $\frac{2}{7}$.
$\frac{7}{7} + \frac{2}{7} = \frac{9}{7}$ or $1\frac{2}{7}$

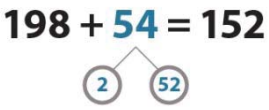


Grade 4 Example 2

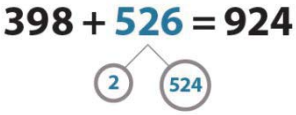
T: 98 + 5 = 100 + ____?
S: 98 + 2 + 3 = 100 + 3.
T: 98 + 5 is ____?
S: 103.



T: 198 + 54 = 200 + ____?
S: 198 + 54 = 200 + 52.
T: 198 + 54 is ____?
S: 252.



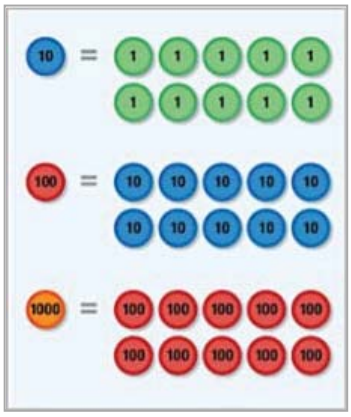
T: 398 + 526 = 400 + ____?
S: 398 + 2 + 524 = 400 + 524.
T: 398 + 526 is ____?
S: 924.

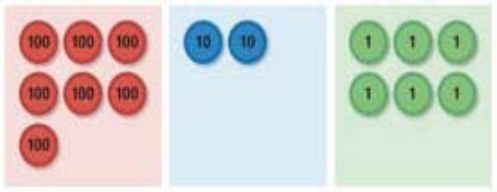


Number Disks

Grade Level **2 – 5**

Description





Place Value Chart with Number Disks

Number disks are non-proportional units used to further develop place value understanding. Like money, the value of the disk is determined by the value printed on it, not by its size. Number disks are used by students through Grade 5 when modeling algorithms and as a support for mental math with very large whole numbers. Whole number place value relationships modeled with the disks are easily generalized to decimal numbers and operations with decimals.

Instructional Strategies

- Play pattern games: "What is 100 less than 253?" Students simply remove a 100 disk and state and/or record their new number.
- Play partner games: Partner A hides the disks from Partner B within a file folder. Partner A says, "I am looking at the number 241. I will make 10 less (physically removing a 10 disk). What is 10 less than 241?" Partner B writes the answer on his personal board/notebook and then states a full response: "10 less than 241 is 231." Partner A removes the folder and the partners compare the written response with the disks.
- Perform all four operations with both whole numbers and decimals on mats.
- Use materials to bridge to recording the standard algorithms for all four operations with both whole numbers and decimals.

Number Line

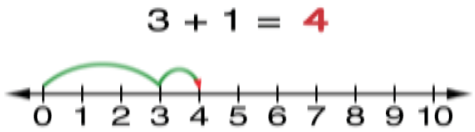
Grade Level K – 5

Description

The number line is used to develop a deeper understanding of whole number units, fraction units, measurement units, decimals, and negative numbers. Throughout Grades K-5, the number line models measuring units.

Instructional Strategies

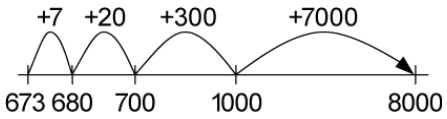
- Measure lengths in meters and centimeters.
- Counting on: Have students place their finger on the location for the first addend, and count on from there to add the second addend.



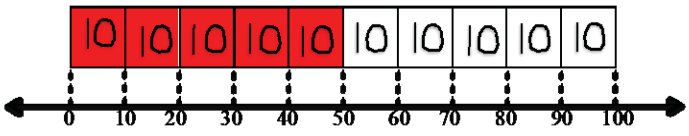
- Have students use a "clock" made from a 24 inch ribbon marked off at every 2 inches to skip-count by fives.

- Compute differences by counting up.

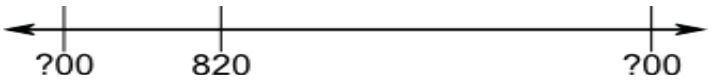
8000 – 673 = 7, 327



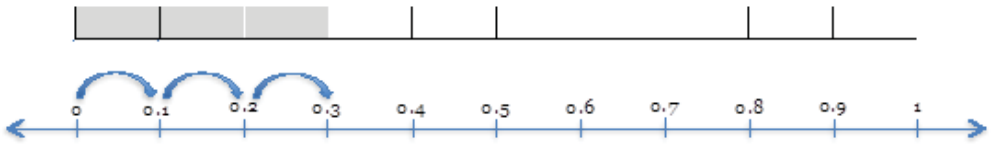
- Multiplying by 10; students visualize how much 5 10's is, and relate it to the number line.



- Rounding to the nearest ten or hundred; e.g., students identify which 'hundreds' come before 820, "820 is between 800 and 900."

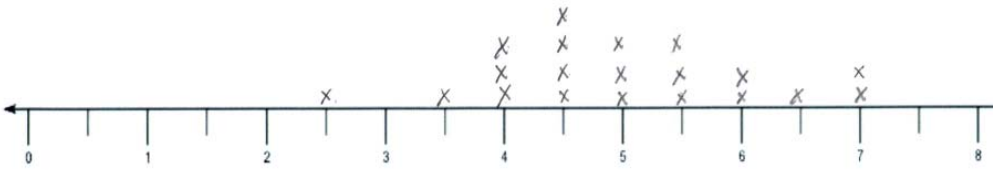


- Model tenths in unit, expanded, fraction, and decimal form.



$$3 \text{ tenths} = 3 \times \frac{1}{10} = \frac{3}{10} = 0.3$$

- Create and analyze line plots.



Number Path

Grade Level **PK – 1**

Description

1	2	3	4	5	6	7	8	9	10

The number path can be thought of as a visual (pictorial) representation of the number tower (see description below) and is foundational to understanding and using the number line. It also serves as a visual representation of 1:1 correspondence and the concept of whole numbers (one number, one space, and each being equal in size). The color change at 5 helps to reinforce the 5 and 10 benchmarks. The number path also serves as an early precursor to measurement concepts and a support for cardinal counting. (If a student places 7 objects in each of the 7 spaces on the path, they must realize that there are 7 objects, not 10. Simply because the path goes up to 10 does not mean there are 10 objects.)

Instructional Strategies

- Sort, classify, and count up to 5 with meaning and then work on extending "How Many" questions up to 10.
- Match amounts to numerals.
- Write numerals 1 to 5.
- Extend the meaning of 6, 7, and 8 with numerals (6 is 5 and 1, 7 is 5 and 2, 8 is 5 and 3.)
- Become fluent with numbers to 10 and practice "before" and "after," as well as relationships of "1 more/less" and "2 more/less"
- Order numbers from 1 to 10.
- Play number order games (e.g., Partner A closes eyes while Partner B covers a number with a penny—then Partner A has to guess the hidden number).
- Fold the number path so that only small sections are visible. Students show 4, 5, 6, 7; teacher says "4, 5, hmm, 7 what number is missing?"
- Play "I Wish I Had" games (e.g., "I wish I had 7, but I only have 5." Student answers by placing a finger on 5 and then counting on to say "2" – the amount needed to make the target number.)
- Match ordered sets with numerals on the number path.

Number Towers

Grade Level **PK – 3**

Description

Number towers, also known as number stairs, are representations of quantity constructed by joining together interlocking cubes such as Unifix ©. In the beginning of the Story, they are used to help younger children quite literally build their knowledge of cardinality by erecting towers of various numbers. Number towers are then used to teach concepts of "more/less" globally and the patterns of "1 more/less" and "2 more/less" specifically. This model leads to an understanding of comparison and the word "than," not only in the context of "more than" and "less than," but also in the context of "taller than," "shorter than," heavier than," "longer than," etc.

Children are encouraged to build towers for quantities 1 through 5 in one color. Quantities beyond 5 are added on in a second color. This color change provides support for several important developmental milestones. First, it facilitates children's understanding of 5 as a benchmark, which provides an important beginning to their ability to subitize. Second, it allows students to see relationships such as "5 needs 2 more to be 7;" "5 is 1 less than 6;" and "5 and 4 is 9, which is 1 less than 10." Finally, it encourages students to count on from 5 rather than starting at 1 to count quantities of 6, 7, 8, 9, and 10.

Such comparisons lead to looking at the parts that make up a number. ("3 is less than 7. 3 and 4 make 7.") These concepts are foundational to students' understanding of part/whole models (see Number Bonds). This, in turn, leads naturally to discussions of addition and subtraction, fact fluencies (+1, +2, +3, -1, -2, -3), and even the commutative property (flip the tower; 3 + 4 or 4 + 3—does the whole change?), which are explored in Kindergarten and Grade 1.

In Grades 2 and 3, as students prepare for and study multiplication and division, each unit in the number stair can be ascribed a value other than 1. For example: "Each of our cubes is equal to three. What is the value of the stair with five cubes?"

3 3 3 3 3

Further, the use of number stairs can be extended to help children understand more complex properties like the distributive property. "Each of our cubes is equal to three. Make a stair with five cubes. Now add two more cubes. The stair with 7 cubes is 2 more threes. So, 5 threes is 15, 2 threes is 6, and together 7 threes is 15 + 6 or 21."

5 threes + 2 threes = (5 + 2) threes

3 3 3 3 3 3 3

Instructional Strategies

- Sort, classify, and count up to 5 with meaning and then begin extending How Many questions up to 10.
- Build a series of towers from 1 to 10, and then use the towers to relate quantities, e.g., "5 is before 6." "6 is after 5." "5 + 1 more is 6." "6 is more than 5." "6 is 1 more than 5." "5 is 1 less than 6." "5 and 2 make 7." "5 + 2 = 7."
- Build a tower that shows 6.
- Build a specific tower and count the cubes. (Cardinality)
- Partners roll dice, each build a different tower and state which has more (less).
- Build a tower while stating the ''one more" relationship (e.g., 4, 1 more is 5).
- Deconstruct the tower while stating the ''one less" relationship (e.g., 7, one less is 6).
- Count on from 5 (e.g., to count 7, students use the color change to say "5, 6, 7"instead of starting from 1). The color change at 5 may be presented to students as a shortcut by having students slide their finger over a group of 5 as they count. (Subitizing)
- Count up from numbers other than 0 and 1.
- Count down from numbers other than 10 to numbers other than 0 and 1.
- Compare numbers within 1 and 10.

Place Value Chart

Grade Level **2 – 5**

Description

Place Value Chart Without Headings
(Used with labeled materials such as disks)

Hundreds	Tens	Ones

Place Value Chart with Headings
(Used with unlabeled materials such as base-ten blocks or bundles)

The place value chart is a graphic organizer that students can use (beginning in Grade 1 with tens and ones through Grade 5 with decimals) to see the coherence of place value and operations between different units.

Instructional Strategies

- Have students build numbers on mats. Place value cards may be used to show the expanded form of a number that is represented on the place value chart.

- Count the total value of ones, tens, and hundreds with any discrete, proportional or non-proportional material such as bundles, base-ten blocks or number disks.
- Model and use language to tell about 1 more/less, 10 more/less on the place value chart with disks when there is change in the hundreds unit.

- Complete a pattern counting up and down.
- Model addition and subtraction using base-ten blocks or number disks.
- Use the mat and place value materials as a support for learning to record the standard algorithms for addition, subtraction, multiplication, and division.

Rekenrek

Grade Level **PK – 5**

Description

20-Bead Rekenrek

100-Bead Rekenrek

The Rekenrek has a 5 and 10 structure, with a color change at 5 (eliciting the visual effect of grouping 5 and grouping 10). The 20-bead Rekenrek consists of 2 rows of 10 beads, allowing students to see numbers to 10 either as a number line on one row or a ten-frame (5 beads on two rows). A 100-bead Rekenrek has 10 rows of 10 beads. Other names for the Rekenrek are "Calculating Frame," "Slavonic Abacus," "Arithmetic Rack," or "Math Rack."

Instructional Strategies

Grades PK – 1

- Count up and down in short sequences (1, 2, 3, 2, 3, 4, 3, 2,…, simulate the motion of a roller-coaster).
- Think of 7 as "2 more than 5."
- See "inside" numbers (subitize – "instantly see how many").
- Count in unit form (1 ten 1, 1 ten 2, 1 ten 3… 2 tens 1, 2 tens 2, etc.).
- Skip-count with complexity such as counting by 10's on the 1's (3, 23, 33, 43, …).
- Group numbers in 5's and 10's. Compare Rekenrek to ten-frame.
- Build fluency with doubles.
- Make 10.
- Add across 10; subtract from 10.
- Build numbers 11-20.
- Show different strategies for adding 7 + 8 (5 + 5 + 2 + 3, 7 + 7 + 1, 10 + 5, 8 + 8 - 1).
- Compose and decompose numbers.
- Solve addition and subtraction story problems (e.g., putting together, taking away, part-part-whole and comparison).

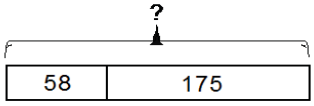
Grades 2 – 5

- Show fluency with addition and subtraction facts.
- Find complements of numbers up to 10, 20, 30, …100.
- Skip count by 2, 3, 4, 5, 6, 7, 8, and 9 within 100.
- Identify doubles plus one and doubles minus 1.
- Model rectangular arrays to build conceptual understanding of multiplication.
- Demonstrate the distributive property. Think of 3 x 12 as 3 x 10 plus 3 x 2.

Tape Diagram

Grade Level **1 – 5**

Description



Rachel collected 58 seashells. Sam gave her 175 more. How many seashells did she have then?

Tape diagrams, also called bar models, are pictorial representations of relationships between quantities used to solve word problems. Students begin using tape diagrams in 1st grade, modeling simple word problems involving the four operations. It is common for students in 3rd grade to express that they don't need the tape diagram to solve the problem. However, in Grades 4 and 5, students begin to appreciate the tape diagram as it enables students to solve increasingly more complex problems.

At the heart of a tape diagram is the idea of *forming units*. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic.

The tape diagram provides an essential bridge to algebra and is often called "pictorial algebra."

Like any tool, it is best introduced with simple examples and in small manageable steps so that students have time to reflect on the relationships they are drawing. For most students, structure is important. RDW (read, draw, write) is a process used for problem solving:

- Read a portion of the problem.
- Create or adjust a drawing to match what you've read. Label your drawing.
- Continue the process of reading and adjusting the drawing until the entire problem has been read and represented in the drawing.
- Write and solve an equation.
- Write a statement.

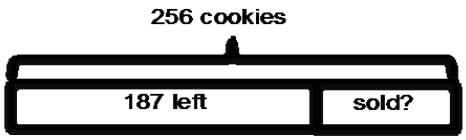
There are two basic forms of the tape diagram model. The first form is sometimes called the part-whole model; it uses bar segments placed end-to-end (Grade 3 Example below depicts this model), while the second form, sometimes called the comparison model, uses two or more bars stacked in rows that are typically left

justified. (Grade 5 Example below depicts this model.)

Rather than talk to students about the 2 forms, simply model the most suitable form for a given problem and allow for flexibility in the students' modeling. Over time, students will develop their own intuition for which model will work best for a given problem. It is helpful to ask students in a class, 'Did anyone do it differently?' and allow students to see more than one way of modeling the problem, then perhaps ask, "Which way makes it easiest for you to visualize this problem?"

Grade 3 Example

Sarah baked 256 cookies. She sold some of them. 187 were left. How many did she sell?

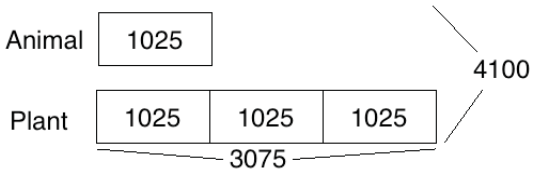


256 – 187 = ☐

Sarah sold ☐ cookies.

Grade 5 Example

Sam has 1025 animal stickers. He has 3 times as many plant stickers as animal stickers. How many plant stickers does Sam have? How many stickers does Sam have altogether?

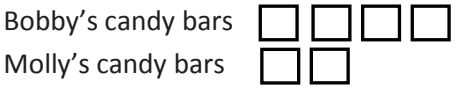


1 unit = 1025
3 units = 3075
2 units = 2050
4 units = 4100

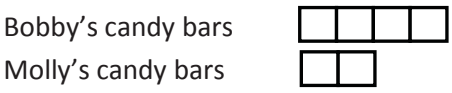
1. He has __3075__ plant stickers.
2. He has __4100__ stickers altogether.

Instructional Strategies

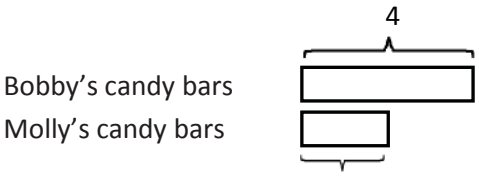
- Modeling two discrete quantities with small individual bars where each individual bar represents one unit. (This serves as an initial transition from the Unifix© cube model to a pictorial version.)



- Modeling two discrete quantities with incremented bars where each increment represents one unit.



- Modeling two quantities (discrete or continuous) with non-incremented bars.

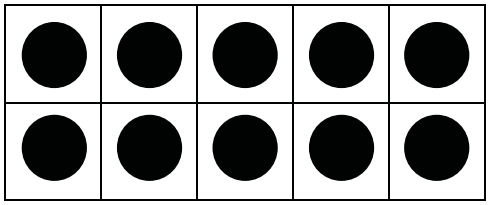


- Modeling a part-part-whole relationship where the bars represent known quantities, the total is unknown.
- Modeling a part-part-whole relationship with one part unknown.
- Modeling addition and subtraction comparisons.
- Modeling with equal parts in multiplication and division problems.
- Modeling with equal parts in fraction problems.

Ten-Frame

Grade Level **PK – 3**

Description



A ten-frame is a 2 by 5 grid (array) used to develop an understanding of concepts such as 5-patterns, combinations to 10, and adding and subtracting within 20. The frame is filled beginning on the top row, left to right, then proceeding to the bottom row building left to right. This pattern of filling supports subitizing by building on the 5 benchmark, as well as providing a pattern for placing disks on place value mats in later grades. Concrete counters as well as pictorial dots may be used to represent quantities on the frame.

In Kindergarten and in early Grade 1 a double ten-frame can be used to establish early foundations of place value (e.g., 13 is 10 and 3 or 1 ten and 3 ones) and can also be used on place value mats to support learning to add double digit numbers with regrouping. The "completion of a unit" on the ten-frame in early grades empowers students in later grades to understand a "make 100 (or 1000)" strategy, to add 298 and 37 (i.e., 298 + 2 + 35), and to more fully understand addition and subtraction of measurements (e.g., 4 ft. 8 in. + 5 in).

Instructional Strategies

- "Flash" a ten-frame for 3-5 seconds then ask students to re-create what was filled/not filled on their own personal ten-frame. (Students may also tell how many they saw or match the "flash" with a numeral card.)
- Use "flash" technique, but ask students to tell 1 more or less than the number flashed.

- Roll dice and build the number on the ten-frame.
- Partner games: Partner 1 rolls a die and builds the number on the frame. Partner 2 rolls and adds that number to the frame (encouraging "10" and "leftovers" or using two ten-frames to represent the sum).
- Play Crazy Mixed Up Numbers. Have children represent a number on the ten-frame, then give various directions for changing the frame (e.g., start with 4 – "two more" – "one less" – "one fewer" – "double it" – "take away three"). This activity has the added benefit of providing the teacher with the opportunity to observe how students count – who clears the mat and starts over each time and who is counting on and/or subtracting.
- Write number stories about the filled and "unfilled" parts of the ten-frame.
- Counting in unit form:

Regular	Unit Form
eleven	1 ten one
twelve	1 ten two
thirteen	1 ten three
twenty	2 tens
twenty-six	2 tens six

- Represent a number between 5 and 10 on the frame with one color counter. Have students add a quantity between 6 and 9 (represented by a second color) to it (e.g., 7 + 6). Encourage students to "fill the frame" and re-state the problem as 10 + 3.